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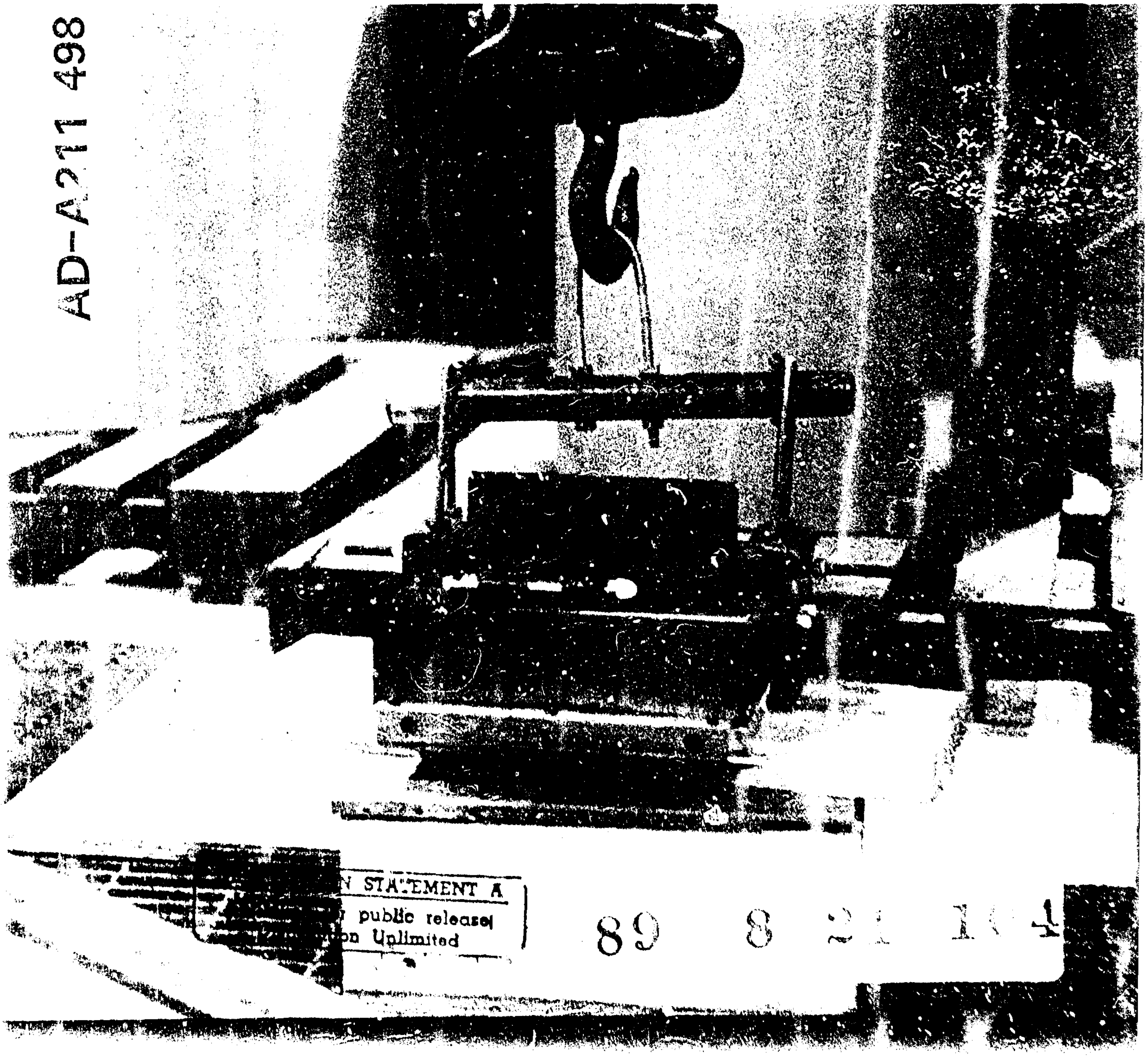
REPORT 89-14

US Army Corps of Engineers
Cold Regions Research & Engineering Laboratory

Dynamic friction of a metal runner on ice

I. Model sled test



For conversion of SI metric units to U.S./British customary units of measurement consult ASTM Standard E380, Metric Practice Guide, *published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.*

Cover: Test sled. (Photo by K. Itagaki.)

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Prepared for
OFFICE OF THE CHIEF OF ENGINEERS

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The effects of runner material and surface conditions on the friction between runners and ice were studied. A model sled was pushed over a 6-m-long ice sheet and the reduction of speed of the sliding sled was measured. The friction calculated by the reduction of speed between two gates indicated that smooth runners showed lower friction at around −1°C than at around −10°C, as expected, but the friction of rough runners showed little temperature dependence. The lower thermally conductive runners showed lower friction than higher thermally conductive smooth runners as the theory predicted, but such effects were obscured on rougher runners.

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PREFACE

This report was prepared by Dr. Kazuhiko Itagaki, Research Physicist, Snow and Ice Branch, Research Division, U. S. Army Cold Regions Research and Engineering Laboratory, Niklaus P. Huber, a student at the Thayer School of Engineering, Dartmouth College, and George E. Lemieux, Research Physicist, also of the Snow and Ice Branch, Research Division, CRREL. Funding for this research was provided by DA Project 4A161102AT24, *Research in Snow, Ice and Frozen Ground,* Task Area C/E1, Work Unit 016, *Adhesion and Physics of Ice.*

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Dynamic Friction of a Metal Runner on Ice
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INTRODUCTION

Anybody who has stepped on ice knows that it can be very slippery. Injuries are commonplace when pedestrians walk on the ice, and poor traction of tires on icy roads has caused many accidents. Nevertheless, this slipperiness also makes it possible to enjoy winter sports and to move heavy cargos by sled. Ice generally allows Amonton's coefficient of friction μ, defined as

$$\mu = \frac{\text{tangential pulling force required}}{\text{normal load}} \qquad (1)$$

to stay constant regardless of velocity or load. For friction between metal and ice near 0°C, μ ranges from 0.005 to 0.01, while most materials exhibit coefficients 10 to 100 times higher.

The most popular explanation for low ice friction is lubrication between the slider and ice by a thin layer of water, formed by the heat of friction or by high pressure. The pressure melting theory was first advocated by Reynolds (1901), while Bowden and Hughes (1939) advanced the friction melting theory. Both theories have been criticized for inadequate experimental support.

Gas lubrication, in the case of magnesium runners, has been proposed by McConica (1950) as an alternative means of lubrication. Niven (1956) stressed that the frictional melting would occur strictly at the tips of the runner's asperities, not on the entire surface of the track drawn by the runner. Tests made by Evans et al. (1976) supported the frictional melting theory. Oksanen (1980) also provided theoretical arguments in favor of this theory.

Some researchers dispute or deny the role of a liquid layer at the ice/slider interface for low friction. Tusima (1977), for example, made tests at very low speeds, allowing him to ignore fluid lubrication. Under such conditions the friction between two solid materials is expressed as the ratio of the force required to shear off the substrate from the ice and the force supporting the load:

$$\mu = \frac{\text{shear adhesive strength}}{\text{compressive yield strength}} \qquad (2)$$

μ can still be low because the shear adhesive strength of ice around the melting point is much lower than its compressive yield strength. Tusima was the first to seriously consider the supporting force of the ice as an integral part of ice friction.

If lubrication by a liquid film is required to reduce drag, liquid between the two solids under high pressure cannot be retained indefinitely. Leakage of the high-pressure water has to be taken into account. Furushima (1972) used a highly simplified roughness profile to calculate the liquid film support and the rate of leaking under ice skates, and found that liquid lubrication would be effective only at roughnesses of less than 0.2 μm or even 0.05 μm.

All the studies discussed so far relate either to theoretical models or idealized testing situations with nearly unchanging conditions. Our intention, described in this report, was to identify basic problems involved in ice friction under dynamic conditions. This would be applicable to understanding the highly dynamic conditions of a bobsled or a skater gliding at high speeds on ice.

An apparatus, including a model sled, driving system and ice sheet, was constructed to simulate a bobsled run, and a number of tests were conducted. The runner roughnesses, ice conditions, and stress level under the runners were chosen to be close to those present during bobsledding, but the available space limited the velocity to less than 1/20 of an actual bobsled's speed.

APPARATUS AND PROCEDURES

General approach

Having in mind the immediate application to bobsledding, we chose the diameter of the runners, the load applied, the ice conditions, and the test configuration to simulate bobsledding as closely as possible within the physical limitation. A small sled weighing 60 kg was propelled by a pneumatic cylinder across an ice sheet grown in a coldroom at CRREL. The coefficient of friction was calculated from initial and final velocity measurements made using optical detectors.

Sled design

The model sled has a pair of runners mounted parallel in grooves in runner holders attached to a 30- × 30-cm square aluminum plate, as shown in Figure 1. A lead- filled box on top of this plate provides nominal contact stress levels under the runners comparable to the real bobsled. A 20-cm-long aluminum plate, mounted with its long edge parallel to the direction of motion, served as an optical shutter for measuring the sled's velocity. The velocity at the gate was calculated from the duration of the interruption of the narrow beam of light by the shutter measured by a photo sensor and an electronic counter. By the measurements of initial V_i and final V_f, velocity between the optical gates separated by distance L of 4.6 m, an average coefficient of friction (μ) was calculated from the energy considerations as

$$\mu = (V_i^2 - V_f^2) / 2gL \tag{3}$$

where g is the gravitational acceleration.

The runners were made of 1/2-in.-round rod material, and both front and back ends were rounded (see Fig. 1). Three runner materials were used: 1010 mild steel, 316 stainless steel, and MP35N multiphase alloy. Standard surfaces used were smooth and rough (0.13- and 1.0-µm centerline average [CLA], respectively), as well as some other values. Bulk runner temperatures was measured at three places—the front end, middle, and tail of the runner—by thermistors and thermocouples inserted in 11-mm-deep holes. Different sets of runners were interchanged by loosening the three bolts holding the runner holders.

Tables 1 and 2 describe in detail the various materials and surface properties used in the tests. The different hardnesses made it difficult to produce identical surface conditions with the different materials. The depth of the grooves cut in the runners with the abrasive paper differed, and the leather strop used for polishing the runners did not produce similar roughnesses on all material surfaces.

Sled propulsion

A 4.5-in-diam Parker-Hannifin pneumatic cylinder was used to accelerate the 60-kg sled to a suitable speed (1.5 m/s) in minimal space. The operator-controlled solenoid valve placed between the accumulator and the cylinder advanced or retracted the ram of the cylinder.

A pair of polyethylene guides attached to the

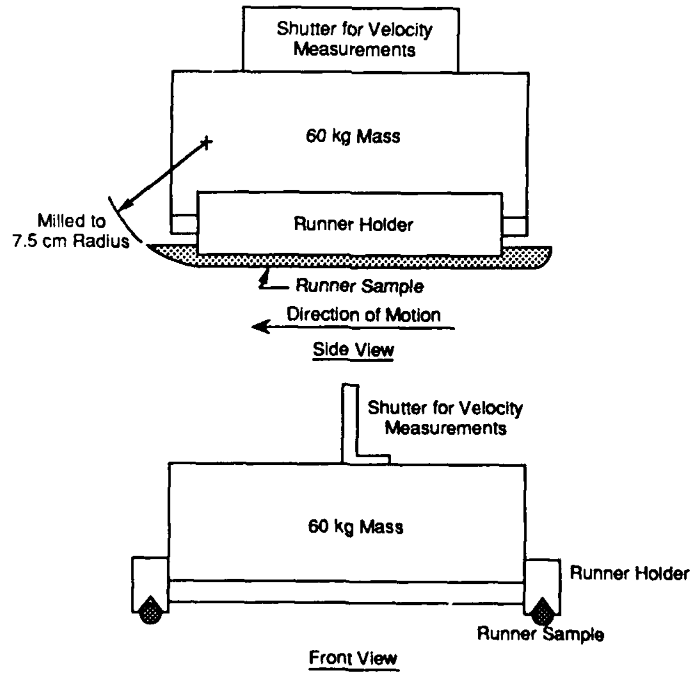

Figure 1. Design of model sled and runners.

Table 1. Properties of runner materials.

	1010 low carbon steel	*316 Stainless steel*	*MP35N*
Runner number	1,2,7,8,14,17	3,4,9,10,16	5,6,11,12,13,15,18
Thermal conductivity (λ)	68 W/m K (CRC)	17 W/m K (CRC)	10 W/m K(Latrobe)
Rockwell A hardness (*H*) (as measured)	51	58	71
Corrosion resistance	Visibly poor.	Good.	Good.
Machinability	Soft and easy to machine.	Intermediate values for hardness and thermal conductivity.	Tough to machine but allows fine surface finishes. Hard enough to retain finishes.

Table 2. Characteristics of runner samples.

Runner number	*Runner material*	*Description*
1	1010 S	smooth (0.125- to 0.2-μm CLA roughness)
2	1010 R	rough (1-μm CLA roughness)
3	316 S	smooth
4	316 R	rough
5	MP35N S	smooth
6	MP35N R	rough
7	1010 HP	more highly polished (0.075-0.1 μm CLA)
8	1010 L	rough, with longitudinal groove
9	316 HP	more highly polished (0.075-0.1 μm CLA)
10	316 L	rough, with longitudinal groove
11	MP35N HP	more highly polished (0.075-0.1 μm CLA)
12	MP35N L	rough, with longitudinal groove
13	MP35N	rough, with an experimental groove
14	1010 LS	rough, but with stropped longitudinal grooves
15	MP35N	rough, with an experimental groove
16	316 LS	rough, but with stropped longitudinal grooves
17	1010	rough, with a magnesium based ski wax
18	MP35N LS	rough, but with a stropped longitudinal groove

ice container directed the first 30 cm of the sled's motion. Although the guides were adjusted very carefully, the course taken by the sled could not be predicted beyond the first 2 m of the run. Only a few runs traced the same track as the preceding run. Soft spots on the ice sheet would deflect the sled, sometimes causing it to hit the wall or even turn around. Data from such runs were rejected.

The sled was stopped at the other end of the ice track by a shock absorber made of an aluminum bar attached loosely to the rams of a pair of smaller pneumatic cylinders. The driving cylinder and the shock absorber were mounted on 5/16-in. aluminum plates that were supported by a frame of angle irons lining the entire box.

Ice sheet preparation

The ice was grown in a plywood box (1.2 × 6 × 0.15 m) (Fig. 2) lined with a sheet of polyethylene. A layer of snow 3 cm deep was spread in the box and saturated with water. After this layer had frozen solid, the procedure was repeated for a second layer. The top of this frozen layer was scraped evenly and a thermocouple was inserted. Five more layers of ice, each about a centimeter thick, were grown in the same way. Each day, the surface of the previous layer was polished with a floor buffer before the water was sprayed on, and the new layer was allowed to anneal overnight at −10°C.

As a final touch, a clean rag saturated with hot water was dragged across the ice surface, resulting in a 1/2-mm layer of ice. This process was repeated after the thin layer froze. Asperities created during testing were removed by buffing with a floor polisher or by scraping manually. Two days were

Figure 2. Ice box with pneumatic cylinder at far end.

allowed for the ice temperature to settle at the desired value before each testing session was begun.

The ice temperature was slightly lower than the coldroom temperature, probably because of sublimation of ice in the dry atmosphere.

Experimental procedure

The hardness and the roughness on the runners were measured three times during the course of the tests. Changes in roughness of the soft 1010 steel were detected and the runners were re-polished or re-roughened to establish the original surface conditions. We also made replicas of the runner and ice surfaces on several occasions for qualitative microscopic observation.

At the beginning of each testing session, the air accumulator for the pneumatic cylinder was charged, and the selected runners were attached to the sled. For some tests, the sled with runners was placed on a heated aluminum plate to warm up, after which the sled was lowered on the ice track with a crane. After measuring ice, air, and runner temperatures, we positioned the sled between the polyethylene guides, and both timers were reset.

The pneumatic cylinder was activated by a three-way switch, which advanced or retracted the ram, or let it idle. The rams of the shock absorbers at the other end of the track needed to be pulled out in order to receive the impinging sled. For each run, the compressed air pressure, the initial and final time intervals, and the runner temperatures (last 33 sets only) were recorded. At the end of a sequence of runs with the same runner, the air, ice, and runner temperatures were recorded again.

RESULTS

General remarks

The model sled was run more than 700 times over the ice sheet using 18 different surface-finished runners made from three materials. These cylindrical runners displayed little directional stability. An imperfection on the ice surface, such as a gouge left by a previous run or a natural asperity, could cause the sled to start sliding sideways, turn around, or even spin uncontrollably. When the sled entered the second optical gate at an angle, the apparent length of the optical shutter was shortened, which caused the calculated speed to be too great. Those results were rejected.

We attempted to obtain at least 10 repetitive readings for each combination of runners, surface treatments and runner temperatures, but some experiments yielded anomalous results that had to be discarded. In total, about 100 runs were rejected as meaningless.

Each entry in Table 3 below represents an average friction value of at least 8 measurements, and sometimes up to 25. Note that the standard deviations (shown under the coefficients of friction) obtained within those sets are encouragingly small. The letters S, R, HP, L, and LS in the code for the material represent the surface finishes—"smooth," "rough," "highly polished," "with a longitudinal groove," and "with a stropped longitudinal groove," respectively—and the number in parentheses is the corresponding runner number, as listed in Table 2.

Effect of runner temperature

In an attempt to find the effect of runner temperature on the coefficient of friction, the runners were heated on a warm aluminum plate before the series of runs. As the runners cooled back down to

Table 3. Ice friction test summary.

Ice T (°C)	Runner T (°C)	1010 S *(1)†	1010 R (2)	316 S (3)	316R (4)	P35N S(5)	MP35N R(6)
−10	−8	0.0123 ±0.0006	0.0113 ±0.0004	0117 ±0.0005	0121 ±0.0004	0116 ±0.0004	0120 ±0.0002
−10	−5 heated	0.0098 ±0.0002	0.0097 ±0.0007	0.0089 ±0.0004	0.0108 ±0.0003	0.0108 ±0.0007	0.0114 ±0.0006
−5	−4	0.0097 ±0.0004	0.0120 ±0.0004	0.0112 ±0.0004	0.0104 ±0.0004	0.0114 ±0.0006	0.0126 ±0.0004
−5	−2 heated	0.0096 ±0.0002	0.0102 ±0.0003	0.0090 ±0.0006	0.0111 ±0.0003	0.0097 ±0.0004	0.0121 ±0.0004
−3	−1	0.0063 ±0.0002	0.0094 ±0.0006	0.0078 ±0.0005	0.0111 ±0.0009	0.0093 ±0.0008	0.0112 ±0.0006
−3	−1	0.0068 ±0.0005					
−3	−1	0.0087** ±0.0010					

Specially treated runners

Ice T (°C)	Runner T (°C)	1010 HP (7)	316 HP (9)	MP35N HP (11)
−10	−8	0.0141 ±0.0004	0.0116 ±0.0003 0.0119 ±0.0003	0.0114 ±0.0004
−3	−1	0.0064 ±0.0004	0.0057 ±0.0004	0.0055 ±0.0003

Ice T (°C)	Runner T (°C)	1010 L(8)	1010 LS(14)	316 L(10)	316 LS(16)	MP35N L(12)	MP35NLS(18)
−8	−7.5		0.0144 ±0.0004		0.0141 ±0.0007		0.0097 ±0.0001
−3	−1	0.0108 ±0.0016	0.0080 ±0.0002	0.0121 ±0.0005	0.0095 ±0.0018	0.0095 ±0.0003	0.0062 ±0.0006

* S: smooth; R: rough; HP: highly polished; L: longitudinal groove; LS: stropped longitudinal groove.
† Runner number from Table 2.
** Comparison runs performed in reverse sliding direction.

ambient temperature during the subsequent runs, a change in friction could be observed. One series of tests was done at relatively high ice temperatures, around −5°C. The effect of temperature change was not clear due to the narrow temperature range and too much scatter in the data. At ice temperatures around −10°C, the results were more meaningful. In Figure 3, the results of all runner temperatures measured on rough and smooth runners are shown. For smooth runners, the higher the runner temperature the lower the friction, whereas for rough runners, the opposite trend was found. Scatter was considerable in some heated runner tests, but a regression analysis over the entire set of data proved that the runner temperature affects rough runners differently than smooth runners.

At the low temperature end, measurements made using the rough runners sometimes yielded lower friction than those for the smooth runners.This result is surprising because we roughened the runners by pressing abrasive paper against the rotating sample so that the grooves were perpendicular to the direction of motion, hardly advantageous from a tribologist's point of view.

Effect of runner surface

Along with the tests with smooth and rough runners described in the previous section, tests were performed with more highly polished runners, runners that had deep longitudinal scratches made in them with coarse sand paper, and runners

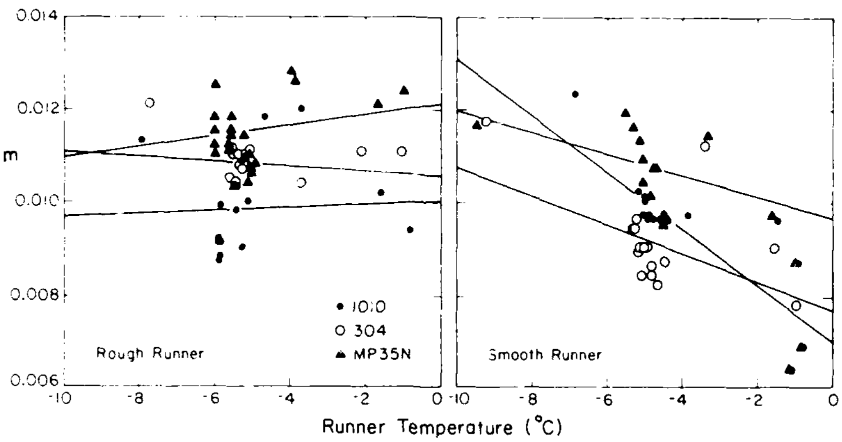

Figure 3. Temperature effect of rough and smooth runners on ice friction.

that were longitudinally scratched and further stropped. The two LS runners were intended to gain more directional control than a perfectly round contact surface would offer. No quantitative measurements of the directional stability were made, but it appears that controllability improved somewhat, though friction increased, particularly for for the runner that was scratched only. Polishing the longitudinally scratched runner surface with the strop and jeweler's rouge (LS in Table 2) reduced the friction considerably. On warmer ice, particularly with MP35N, friction was lower after this treatment than when the runner was polished as usual, and friction was nearly as low as for the highly polished, stropped runner. Comparison of these friction data is difficult since roughness measurements were difficult even with the use of an advanced profilometer and also since the roughness changed during the test on softer material such as 1010.

The surface conditions of the runners may have been affected by abrasive dust particles contained in the ice grown in this coldroom. Tests usually involved runners sliding at least 100 m on the ice sheet. Surface replicas taken at the front end of a runner made of 1010 mild carbon steel (see Fig. 4), after about 50 m of test runs, show that soft materials such as this will suffer substantial abrasion.Therefore, surface conditions may not have been the same even in one series of the tests, partic-

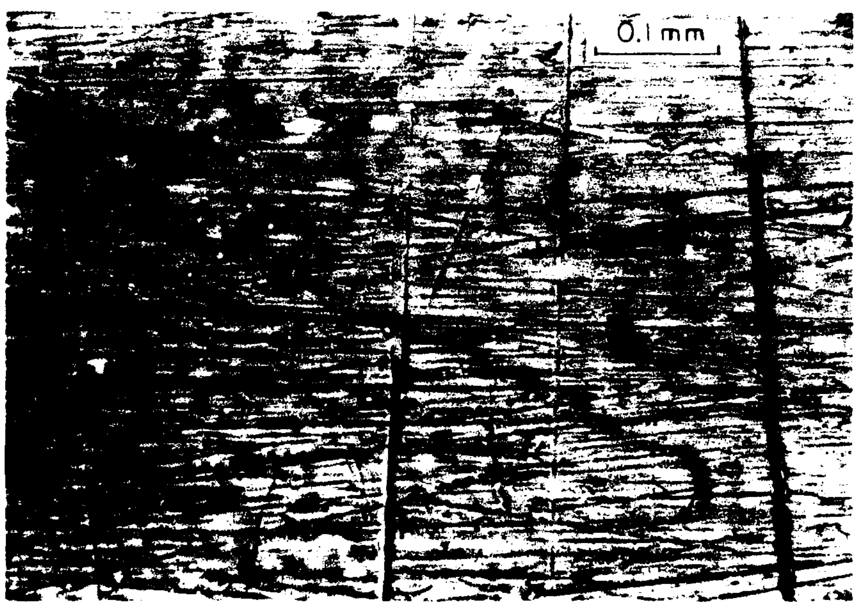

Figure 4. Formvar replica of 1010 mild steel runner after 50 m of test runs.

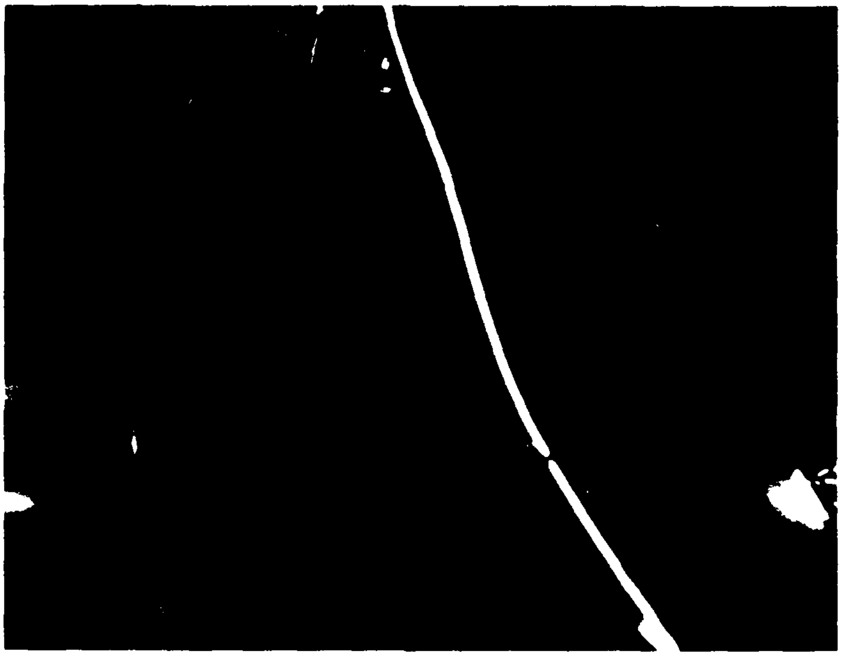

Figure 5. Trajectory of sled run on the ice surface.

ularly for the soft materials.The surface conditions eventually became consistent after several series of tests, but the runner no longer had its original roughness.

Effect of ice surface conditions

As discussed, letting the sled run straight in the test section of the ice box was no simple matter. Even though meticulous adjustments of the guides were made at the driving section, the sled tended to stray away from pre-existing tracks. Several places on the ice sheet were particularly weak and tended to be crushed. Such surface irregularities apparently became more pronounced with repeated passage of the runners. Occasionally, the runners stayed in the same track for several runs, but most strayed away from the previous track after the first 2 m of the run.

In order to see the motion of the sled during the run, we fastened a small flashlight to the sled and turned off the lights in the coldroom. A series of photographs were taken by keeping the shutter of the camera open during the run. The effect of surface irregularities of the ice is shown in Figure 5, in which the trajectory can be seen to deviate several centimeters from a straight line. The impact of such irregularities on the overall coefficient of friction cannot be determined from those observations, but a reduction of friction is unlikely. Breaking asperities or crushing weak parts of the ice surface requires energy, implying an increase in friction.

Effect of velocity

The initial velocity of the sled was dependent on the pressure of the supplied air, and no efforts were made to control this velocity. The resulting range of initial velocities, from 1.2 to 1.6 m/s, is not broad enough to yield reliable information on the effect of varying the velocities.

Effect of runner materials

According to the frictional melting theory, materials with lower thermal conductivities should retain more of the heat generated by friction, thus melting more ice and producing a thicker lubricating water layer that would result in lower friction. Contrary to these expectations, MP35N, which had the lowest thermal conductivity among the three materials we used, generally showed the highest friction when polished as usual (S) and when roughened (R), as illustrated in Table 3 and Figure 3.

When the runners were highly polished (HP), the prediction of better gliding for runners with lower thermal conductivity held both at −10°C and at −3°C, as shown in Table 4 (compiled from the data in Table 3). MP35N, with the lowest thermal conductivity, showed the least friction in the highly polished condition, followed by the slightly more

Table 4. Selected data: effect of runner materials.

Prep.	*Material*	*−3°C*	*−10°C*
Polished	MP35N	0.0055 ±0.0003	0.0114 ±0.0004
(HP)	316	0.0057 ±0.0004	0.0116 ±0.0003
	1010	0.0064 ±0.0004	0.0119 ±0.0003
Smooth	MP35N	0.0093 ±0.0008	0.0116 ±0.0004
(S)	316	0.0078 ±0.0005	0.0117 ±0.0005
	1010	0.0063 ±0.0002	0.0123 ±0.0006

conductive 316 stainless steel and by the highly conductive 1010 low carbon steel. However, the difference is insignificant, except for the case of the smooth runner at −3°C, when the standard deviations are taken into account. In the −3°C case, the trend is reversed; the lowest friction coefficients obtained in the entire study came from tests at −3 °C using highly polished MP35N runners. These results suggest that very smooth surfaces may be needed to reveal the effects of the runner materials' thermal conductivities.

During most of the tests comparing runners with the usual degree of polishing, 316 stainless steel showed the least friction, except at −3°C, where 1010 showed the lowest friction coefficient. Among rough runners, 1010 performed best, while MP35N yielded the highest friction coefficient.

This ranking may have resulted from asperities on the runner surface, which would increase the friction. These asperities are most easily abraded on a softer material such as 1010 or even 316, rather than an extremely hard one such as MP35N. One would thus expect a gradual lowering of the friction in rough runner tests with the soft 1010. However, such a trend is not clear in our data.

Observations of replicate surfaces

The vicinity of the contact areas both on the runners and the ice were observed using replication techniques. The ice surface was replicated with a 5% ethylene dichloride solution of Formvar (see Fig. 6). Very fine grains of recrystallized ice were observed in replicas of the ice immediately under the track of the runner. Apparently heavy deformation generated a heavily strained layer, and then quick grain growth took place by an annealing process before the replicas were made. On both sides of the runner track, an even finer structure, probably made either from ejecta of refrozen spray of high pressure liquid under the runner or tiny crushed ice particles, formed parallel lines about 50 μm away from the edges.

Statistical analysis

The wealth of data gathered during the sled's 700 runs was first analyzed with a polynomial regression package by using the temperature, material properties and surface finish as parameters. The statistical significance of the parameters thus obtained, such as ice temperature and runner material and finish, was found to be of little meaning since rough runners showed opposite temperature effects than smooth runners, as shown in Figure 3. Therefore the results for in-

Figure 6. Formvar replica of ice surface, showing heavy damage on the surface

Table 5. Linear regression equations.

Smooth runner	
1010	$\mu = 6.956 \times 10^{-3} - 6.039 \times 10^{-4}T$
316	$\mu = 7.669 \times 10^{-3} - 3.019 \times 10^{-4}T$
MP35N	$\mu = 9.649 \times 10^{-3} - 2.286 \times 10^{-4}T$
Rough runner	
1010	$\mu = 1.009 \times 10^{-2} + 3.899 \times 10^{-5}T$
316'	$\mu = 1.006 \times 10^{-2} - 4.606 \times 10^{-5}T$
MP35N	$\mu = 1.198 \times 10^{-2} + 8.200 \times 10^{-5}T$

dividual material and finish were analyzed separately.

The results are summarized in Table 5 and the regression lines are drawn in Figure 3. Although thermal conductivity ranged from 68 W/ m K for 1010 steel to 10 W/m K for MP35N, little difference was observed for nominally smooth finished runners. Sometimes runners made of higher thermal conductivity material (1010) showed lower friction than those of lower thermal conductivity material such as MP35N, contradicting the thermally controlled friction theory. An interesting trend is that smooth runners showed a strong negative temperature dependency in which the higher the temperature, the lower the friction. (However, rough runners showed little temperature dependency.)

DISCUSSION

The surface temperature of ice during bobsledding is within 95% of the absolute temperature of melting ice even on relatively cold days (–20°C). The surface characteristics of the ice may be changed by a small change in temperature since it is so close to the melting point. Frequently ice surfaces are said to be covered by a "liquid-like layer" above –10 °C (Faraday 1859, Weyl 1951, Nakaya and Matsumoto 1954, Fletcher 1962 and 1968, Jellinek 1967, Valeri and Mantovani 1978).

Ice friction is a complicated physical process. At low temperatures and slow speed, the frictional force appears largely to result from solid-solid surface interaction. Even under such conditions, ice friction generally is very low (Tusima 1977). As the ambient temperature approaches melting, solid-liquid interaction comes into play together with the resistance caused by ploughing ice and molecular adhesion between the ice and solid substrates. Thick liquid layers may not be necessary for lower friction even when the liquid lubricating layer controls the friction. With a thicker liquid film, it is easier for the water to squeezed out; thus more energy is needed to melt more ice to maintain the thickness of the film. Various calculations (Bowden and Hughes 1939, Bell 1948, Furushima 1972, Evans 1976, Oksanen 1980), assuming the bulk viscosity of water for the lubricating layer, have indicated that the layer may be as thin as a few nanometers or tenths of a micrometer. However, a very thin water layer bounded by solids may show considerably different characteristics (Hori 1956), and so the validity of such an assumption could be doubted.

In order to optimize the performance of bobsled runners, we must understand the contributions of drag mechanisms under various conditions. Knowing the contributions of the mechanisms, we can optimize the runner materials, surface preparations and runner shape correctly. We may also gain insights into the controllability problems and perhaps suggest ways to remedy them.

There are several conceivable mechanisms that could absorb energy from the running sled. Aerodynamic force would predominate at speeds higher than 20 m/s. This is a difficult area, involving not only the bobsled shape but posture and clothing of driver and crew, interference from walls and ice surfaces, etc.

For simplicity of discussion let us assume that, on the first half of the course, the bobsled runs at half the speed under friction control, and then runs the second half twice as fast as the first under aerodynamic control. The time spent in the first half is twice as long as that of the second. Therefore, an improvement of 1 % in the first half is twice as effective as in the second, in terms of time. In order to recover a 1 % loss in the first half, aerodynamic drag has to be reduced 2%.

Several energy loss mechanisms are involved in the slower section and some are part of friction/ lubrication effects. They are 1) molecular interaction, 2) mechanical deformation, 3) thermal energy dissipation, and 4) hydrodynamic processes. Each of these mechanisms can be further divided as shown in Table 6.

The potential energy difference between the start and goal of the Mt. Van Hoevenberg bobsled run in Lake Placid, New York, for a four-man bobsled (with a maximum allowable weight of 630 kg) is roughly

$$630 \text{ kg} \times 9.8 \times 150 \text{ m} = 926{,}100 \text{ J}$$

where 150 m is the vertical drop of the run. Meanwhile the kinetic energy left at the goal, assuming a final velocity of 35 m/s, is

Table 6. Energy loss mechanisms for ice–runner interactions.

Molecular interactions
Solid–solid interactions occurring when ice makes direct contact with the runner, involving adhesion and sliding.
Solid–liquid interaction causing boundary layer lubrication.
Viscous drag dominating within liquid, if it exists.

Mechanical deformation
Elastic deformations of ice eventually transmitted to the ground and air as sound and vibrations.
Plastic flow of ice, which is important to accommodate the runner on the ice surface and is the major process to make a groove on the ice surface.
Shaving and crushing of ice.
Crack formation.

Thermal energy dissipation
Melting,which provides a lubricating fluid layer.
Heat conduction, which is presumably the major factor controlling fluid layer thickness.

Hydrodynamic processes
Viscous and turbulent drag within the fluid layer.
Squeezing out of fluid under pressure, which reduces the thickness of the lubricating layer.

$$630 \text{ kg} \times (35 \text{ m/s})^2 /2 = 385{,}875 \text{ J}$$

which means that about 42 % of potential energy is still left as a kinetic energy.

The energy loss by friction in the 1500-m course, assuming a frictional coefficient of 0.01, is

$$630 \text{ kg} \times 9.8 \times 1500 \text{ m} \times 0.01 = 92{,}610 \text{ J}$$

or 61.74 J/m.

In order to compare these calculations with our results, the value was converted into units of joules/kilogram-meter. Then the value becomes 0.098 J/kg m, which is 10 % of the potential energy.

We can calculate the rate of energy loss L by friction μ per mass per distance another way as

$$L = \mu g \text{ J/kg m}$$

which is again 0.098 J/kg m, assuming that μ = 0.01. The energy loss comprises the mechanisms described above and perhaps some more. Let us examine the contributions of the individual mechanisms.

Molecular interactions

Direct contact between runner and ice

This energy loss is one of the most difficult to estimate. If we accept the values obtained from very slow sliding experiments made by Tusima (1977), μ would be about 0.02 at a sliding speed of 0.1 cm/s (0.196 J/kg m) or faster, though the value increases as velocity decreases. Our experimental values never exceeded 0.015 or 0.147 J/kg m, indicating that a considerable part of our runner surfaces must have been in lower friction. Most likely they are lubricated by fluid. If we assume that only direct contact between the ice and runner is responsible for the measured friction, 0.015/0.02 = 75% of the runner was in contact with the ice and the rest provided support but no friction at all. In a real case, probably the direct contact area is much smaller and other mechanisms provide support of the load and share frictional force.

Solid–liquid interaction

Both runner–fluid and ice–fluid interactions cause boundary layer lubrication but the extent of their contribution is very difficult to examine. Various studies on ice friction indicated that the total thickness of the fluid layer is less than 1 µm and maybe even 5 nm, which is about 10 molecular layers of thickness. Tozuka et al. (1979) observed regelation and concluded that to advance a wire through the ice requires a 0.5-µm-thick water film covering the wire.

Mechanical deformations

Vibration

Energy loss by vibration of the ground and generation of sound through elastic deformation will vary with ice surface roughness, sled speed and other factors, such as the shape of the runners.

Comparing the loudness of the noise with the public address system at the Mt. Van Hoevenberg, the acoustic energy output from the sled would be somewhere between 10 to 100 W. At the bottom of the course, the bobsled reaches a velocity of 35 m/s. Energy loss by acoustic noise would be about 100/35 = 2.86 J/m or 0.0045 J/kg m. This is about 5 % of the energy loss by friction, as estimated before. Loss by vibration of the ground is difficult to estimate.

Ice carving and crushing

The tracks carved by the runners were less than 1 mm wide in the present experiments. The track carved by a real bobsled at a sharp turn may be much wider and deeper. Let us assume that these grooves were made by plastic flow by indentation with the runner as indenter. The energy is supplied by the drop of 60 kg of sled mass. The depth of the 1-mm wide indentation made by 6-mm radius indenter would be

$$6 - (6^2 - 0.5^2)^{1/2} = 2.1 \times 10^{-5}\,\text{m}.$$

Therefore the energy supplied by a 60-kg weight is

$$60 \times 2.1 \times 10^{-5} \times 9.8/2\,\text{J} = 6.2 \times 10^{-3}\,\text{J}$$

The energy required by a 25-cm-long runner to carve a 1-m-long groove is $4 \times 6.2 \times 10^{-3}$ J or 4.1×10^{-4} J/m kg. If the same value is applicable to a real bobsled, then $4.1 \times 10^{-4}/0.098 = 0.42$ % of the frictional energy loss goes to carving the tracks.

If we also assume that the groove having a cross-sectional area 0.01 mm^2 was formed by the 60-kg sled shaving ice with a crushing strength of 100 kg/cm^2 (Butkovich 1954), the energy loss would be 1.67×10^{-4} J/kg m.

Crack generation

In order to generate a crack, a certain threshold stress is required. Present test conditions seem just under such a threshold. Under the simple cracking conditions, the energy required to create a new surface (newly created surface area × surface energy) would be much smaller than the release of stored strain energy. The total surface area of the crack would not exceed 1 mm × 1 m = 0.001 m^2. The surface energy value obtained by Ketcham and Hobbs (1965) was 109 ergs/cm^2 or 0.109 J m^2, so that the energy expended is at most 0.000109 J m if cracks were formed. Since crack formation energy is a nonlinear function of load, we cannot compare it directly with other values, but as our tests did not generate extensive cracks, the surface formation energy should be negligible. Probably the release of elastic stress at the crack can be much larger than the energy required for crack surface formation, but such a value is difficult to estimate.

Thermal energy dissipation

Melting of the ice

Melting ice to make the track can be very energy intensive. The width of the track made by the 60-kg sled was about 1 mm. The cross-sectional area of the track having 6-mm radius is roughly 0.01 mm^2. To make a track of this size by melting, 14.25 J/m or 0.057 J/kg m of energy is required. This is more than 50 % of the estimated energy loss by friction. This value is about 3,000 times larger than that expended during the ice crushing process described above.

Heat conduction

Evans et al. (1976) calculated the lubricating layer thickness in their experimental results by assuming that fluid lubrication is the sole friction mechanism and obtained the thickness of 5 nm, which is too small. They developed a theory with mixed lubrication mechanisms by assuming that the various thermal energy loss mechanisms had adjusted by themselves to produce the observed results. Using this theory, Evans et al. (1976) obtained a more reasonable 0.3 µm, but did not provide any figure for the runner surface roughness. If we accept their theory, their runner surface roughness would have needed to be less than 0.3 µm, which indicates a surface polished to somewhat close to mirror-like finish. Furushima (1972) calculated the thickness of the lubricating fluid layer as a balance between melting of ice and loss of high pressure fluid from the edge of the runner and estimated it to be about 0.1 µm. Therefore we would expect to see the effect of thermal conductivity on the runners having a roughness of less than about 0.1 µm, and our observations of low friction with highly polished, low thermally conductive runners seem to confirm this. None of those arguments includes interfacial water properties, but these are assumed to be the same as those of the bulk water.

Hydrodynamic processes

Since fluid seems to exist between runner and ice (Tusima and Yosida 1969), viscous and turbulent drag within the fluid layer between the ice and runner could contribute the energy loss. However, a turbulent condition is unlikely to exist since the thickness of the fluid layer is less than 1 µm. The Reynolds number is at most 40, which is far smaller than the transition value of 2000. Only energy loss caused by viscous drag would contribute.

As discussed before, various calculations indicated that the thickness of the liquid layer beneath the model sled's runners ranged from 5 nm to less than 1 µm, if we assume that the viscosity value of bulk water at 0 °C to obtain the observed coefficient of friction by viscous drag only. One flaw in those calculations (with the exception of Furushima's) is that they assumed the water generated under the runner either by pressure melting or frictional heat never leaked away. Another unrealistic assumption was that the runner surface was perfectly smooth and flat. In reality, protrusions on the rough runner surface would make direct contact with the ice, increasing drag.

Leaking of pressurized liquid depends on the runner and ice surface roughness, as well as the configuration of the runner. If the runner was hollow ground, as are skate blades, leakage from the side of the blade would be considerably reduced because the sharp edge would effectively seal off the leaking liquids. Of course, rough ice surfaces would reduce the sealing effect drastically. For the round runner used in bobsleds, fluid leaks more easily so that the effect of fluid lubrication would be reduced. Direct contact would prevail, with surface smoothness more important.

Comparison of energy loss mechanisms

Of the total potential energy of 926,100 J, about 30 to 40% is still left at the end of the run as kinetic e ergy loss. Frictional energy loss accounted between 10 to 30% of the total energy. Most of the remaining would be spent by aerodynamic drag. The major energy loss mechanisms discussed above can be broken down as shown in Table 7.

Table 7. Energy losses for each mechanism.

Mechanism	*Energy loss (J/kg m)*
Total energy loss	0.098*
Direct contact	< 0.196
Vibration	4.5×10^{-3}
Indenting track	4.1×10^{-4}
Groove by crushing	1.67×10^{-3}
Crack formation	$< 1.09 \times 10^{-4}$
Groove by melting	0.057
Heat conduction	> 0.47 (see text)

* Assuming $\mu = 0.01$.

Of the seven energy loss mechanisms considered here, direct contact, groove formation by melting and heat conduction contribute significantly, while the other four can be disregarded. The direct contact term is a function of the real contact area and can be much smaller than the apparent contact area. Dissipated energy will eventually be converted into heat and constitute part of heat conduction term. Part of the heat used for melting will bring the ice surface temperature to the melting point and then be dissipated through heat conduction. Most of the heat used to melt ice will be released and carried away as ejecta along the track.

Evans et al. (1976) estimated the contribution of heat conduction through ice on µ to be 0.048 or 0.47 J/kg.m. Heat conduction through the runner would contribute further, making this term far larger than energy loss by friction. However, some assumptions used by Evans et al. for their calculations could be questioned. Leakage of meltwater, for instance, was neglected but should be carefully treated. The heat of melting seems underestimated and the properties of meltwater were assumed to be the same as those of the bulk water. A major source of discrepancy may be in the assumption that ice in all the contact area is at the melting point of T_m. This assumption contradicts their previous calculation of sufficient water film thickness to produce the measured friction of 5 nm and discards completely the water film lubrication hypothesis. Probably we should assume that a considerable portion of the runner makes direct contact with the ice so that at least the temperature at the direct contact points is much lower than T_m for the thermal energy loss calculation.

Many theories and experimental results have been published, covering a wide range of materials, coatings, configurations, temperatures and stress levels on various types of ice, ranging from single crystals to urea-doped ice. In Table 8, a list of results for freshwater ice and metal sliders is shown. In column 6 friction coefficient results of various authors made under conditions comparable to those of the present study are listed. It is interesting that the present results and those by Kobayashi et al., both having a similar apparatus configuration, formed the lowest friction group, with 50 to 100% lower values than the others.

Two major differences between our (and Kobayashi's) tests and the others that may cause lower friction can be pointed out. One is that the runners went through a generally fresh, frozen ice surface, unlike the other tests in which the runner repeatedly contacted machined or rubbed surfaces.

Table 8. Recent ice /metal friction studies.

Authors	*Configuration*	*Speed (m/s)*	*Material*	*μ range*	*Comparable conditions (μ)*	*Press. level (MPa)*	*Roughness (μm CLA)*
Niven (1954)	Slider on ring ice	0.9	Stainless steel	0.19 @–17°C 0.003 @–2°C*		0.11 5.4	
Schulz & Knappworst (1968)	Slider on ice disk	0.001 to 0.02	Stainless steel	0.8 @ –150°C to 0.01 @ 0°C	NA	0.03	~1 ?
Kobayashi et al. (1970)	Skate on skate rink ice	0 to 1.5	Steel skate	0.0042 @ –2° C 0.0102 @ –10 °C	0.0042 0.0102	>0.5	
Evans et al. (1976)	Slider on ice cylinder	1 to 15	Steel, copper perspex	0.01 ~ 0.03 @ –11.5°C	0.02	5 to 20	
Grothues-Spork (1977)	Cone in matched hole	0 to ?	Steel	0.07 @ –20 °C	NA	0.05 to 3.5	
Tusima (1978)	Slider on single crystal	0.0001	Steel Tungsten carbide	0.15 @–21°C 0.03 @ –10°C	NA	60~90	< 10
Oksanen (1980)	Sector slider on ring ice	0.5 to 3	Steel & others	0.01 @ –1°C 0.03 @ –15°C 0.01 @ –1°C 0.058 @ –15°C	0.01 to 0.02	0.00087 to 0.00435	Acid treated & sand blasted
Calabrese et al. (1980)	Annular slider on ring ice	0 to 1	S.S, Al Plastic coatings	0.1 @ –3 to –20°C	0.1	0.3	1 ~ 6.1
Spring et al. (1985)	Sector on ring ice	0 to 13.5	Steel	0.017	0.017		
This study	Sled on ice sheet	1.5	Steel S.S M.P. alloy	0.005 @–2°C 0.015 @ –10°C	0.005 0.015	>1.5	0.5 to 2

* Load vs drag force was nonlinear. This is a typical value under comparable conditions.

Possibly, repeated rubbing of the ice by the same area of the runner would make a larger direct contact area. The other reason is that the pressure level of our studies and Kobayashi's may be near optimum. Both our studies and Kobayashi's (1970) were made around 1 MPa while others were in the kPa range [Shulz and Knapport (1968), Oksanen (1980), Spring et al. (1985)] or 10 to 100 MPa [Evans et al. (1976), Tusima (1978)]. Oksanen (1980) reported the trend of a decreasing coefficient of friction with increasing pressure in the kPa range. Obviously, extremely high pressure would crush the ice so that the apparent coefficient of the friction would become higher. The pressure level of hundreds of kPa to 1 MPa could be the optimum pressure level. Of course, such an optimum pressure level would be a function of temperature. It is a common practice in bobsled racing to select runners of different curvature to best fit the temperature at race time. The best fit curvature of the runner would result in optimum pressure for the race conditions.

The most popular mechanism to explain the low frictional coefficient ice is the lubrication of liquid melt between the ice and slider. Tusima (1977, 1978), however, has experimentally shown that the coefficient of friction is still low even under conditions when no fluid can be expected. Ice, at temperatures we usually encounter, say between 0° and –30°C, is very close to melting point (a homologous temperature of 0.9 the melting temperature). Within such a temperature range the surface and interface can show anomalous features. For example, Mantovani et al. (1980) reported the existence of a viscous layer between ice and a strain gage frozen onto the ice.

Such a thin layer of water may show quite different properties than bulk water. Hori (1956) reported that a layer of water less than 1 µm thick, sandwiched between two optically flat glass plates, can be supercooled below –100 °C. Sometimes such layers show very strong resistance to shear forces. Occasionally the cohesive strength of such layers at room temperature exceeded the strength of glass so that Hori was not able to separate the glass plates without breaking them. This is comparable to the difficulty in trying to separate stacked microscope slides.

Itagaki and Tobin (1973) observed the surface mass transfer of ice by the groove decay method and concluded that at least 68 %, and more for the longer wavelengths, of the decay is contributed by the viscous flow. The factor for the viscous flow appeared to be

$$F = 4.29 \times 10^{-10} \text{ m/s}.$$

We can calculate viscosity from this factor since $F = \gamma_s / 2\eta$ and the result is 1.27×10^8Pa s. This value is 7×10^{10} times larger than the viscosity of water at 0°C of 1.73×10^{-3}. Calculation of film thickness in the same way as Evans et al. resulted in an outrageous thickness of 3.25 m. Probably such a result is caused by the assumption made by Mullins (1959), on which the Itagaki and Tobin values are based, for calculating the contribution of viscous flow by flattening the surface. Mullins assumed that the whole body, not the surface film, of the solid is a viscous substance. Though the value may be incorrect, the possibility of such an anomalous layer remains.

A rather surprising logical consequence may appear if we assume that the viscosity of normal water causes the friction. Since viscous drag is inversely proportional to the thickness, the contribution of viscous drag to the overall friction rapidly diminishes as the thickness of the water film increases. Up to a few nanometers of thickness, a film could appreciably contribute to the friction. With a 1-µm-high protrusion, the viscous drag contribution would be limited to the very close vicinity of the real contact area, which would probably be almost indistinguishable from the real contact area. Therefore, most of the water existing within the contact area would contribute only to support of the load and not to the friction.

If we adopt fluid lubrication as the mechanism to explain the low friction of ice, high pressure water trapped between the runner/slider and ice should leak around the boundary of the contact area. Furushima's calculation (1972) indicated that under the usually observed sliding conditions of an ice skate, the thickness of the water layer would be of the order of 0.1 µm. Therefore pure liquid-lubrication-controlled sliding can occur with a runner surface roughness of 0.1 µm or less. This is almost a mirror finished surface. Sliding on a surface rougher than this would result in a mixture of fluid lubrication and direct contact sliding. When the sliders stopped, the ice surface would gradually conform to the runner surface so that, in re-starting, the slider would need to provide sufficient force to break off such rough ice, together with purely adhesive bond breaking.

SUMMARY AND CONCLUSIONS

The experiments using the model sled described in this report provided us with information about the basic problem area of ice friction. It was unfortunate that only a limited range of velocities could be achieved, as a significant dependence of friction on velocity is documented by others. Dynamic problems such as vibrations would be more pronounced at a higher speed, but the effect of these on the results presented here cannot be predicted reliably. One should exercise caution before applying the results of this study to bobsledding.

Tests were conducted at ambient temperatures ranging from –10° to –3°C with runner samples made from three different metals with various surface profiles. The coefficients of friction were highest at the lower temperatures, corresponding to the expectation that less fluid lubrication and more solid/solid interaction is unfavorable for gliding.

At an ice temperature of –10°C, we noticed that runner temperature affected smooth runners differently than rough ones. Specifically, heating seemed to benefit smooth runners (0.125- to 0.2-µm CLA roughness), but increased the resistance for rough (1-µm-CLA) runners. An explanation for this dichotomy may be that smooth runners use the additional heat to melt a thicker lubricating layer, whereas the asperities of the rough runners bite too much into the ice when heated, increasing the friction.

The lowest coefficients were recorded at ice temperatures around –3°C, but the ice surface became too difficult to maintain near the melting point to allow further experimentation. The highly polished runners (0.075- to 0.1-µm-CLA roughness) encountered exceptionally low resistance, with coefficients ranging from 0.0055 for MP35N to 0.0064 for 1010 mild steel.

Unlike the less smooth runners, highly polished runners behaved very much according to expectations raised by the frictional melting theory. Materials having lower thermal conductivity showed the least resistance, presumably because the heat generated by friction was conducted away more slowly from the contact area and the thicker lubricating layer could be retained. Furushima's theoretical calculations (1972), which predicted diminished lubricating effects if the surface roughness average exceeds 0.05 to 0.2 µm CLA, seem to hold here.

The findings presented here will be verified in a more thorough study. A more versatile apparatus allowing us better control of the runners will be used for simulations of the more dynamic conditions encountered at higher velocities.

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